Income-Share Agreements on the Job Market: Debt Versus Equity^{*}

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Abstract

Income-share agreements (ISAs) recently have been gaining traction as a way for students to finance college education, marketed as a way for students to reduce the down-side risk of winding up in a low-paying job with high student debt. Because ISA payments are a fraction of the on-the-job wage, incentives for both applicant and provider are different from a traditional debt-financed job applicant on the job market. I develop a labor-search model to show how financing affects job-market outcomes such as wages, search duration, and overall utility, set within an equilibrium framework in which the terms and methods of financing are endogenous. I show that ISAs can constitute an important part of the college-financing decision for financially-disadvantaged potential college students, and can act well as a substitute for traditional debt-markets when the cost of college is neither very low nor very high.

Keywords: economics of education, income sharing, labor search, student debt, paying for college, student aid

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1 Introduction

Income-share agreements—agreements whereby students receive money for tuition and fees in return for a share of their future income after graduation—have grown rapidly in popularity over the last few years. Since first conceived by Milton Friedman in 1955, income-share agreements (ISAs) occasionally have been used as a part of the financial aid landscape in higher education, but never more so than since 2016 when the first large-scale ISA of the modern era, the Back a Boiler-ISA Fund, began at Purdue University. Since then, similar programs have begun across the United States, providing students with an alternative means of financing their collegiate educations.

In this paper, I use a simple microeconomic model to analyze the effect of income-sharing on numerous labor market outcomes. In particular, I find that ISAs put upward pressure on wages but may increase or decrease take-home pay, with increases in take-home pay being associated only with a form of ISA that directly helps students find better jobs. I additionally show that ISAs tend to increase the time it takes for applicants to find jobs, with another exception for the ISAs that invest in students' labor market outcomes. I find sufficient conditions for both positive results to hold for these types of agreements. These types of investment-providing ISAs may also expand access to higher education for students facing particularly high costs of attending. Students are allowed to choose their source of funding endogenously, and I find that in the optimal financial-aid environment, all students utilize income-share agreements to some degree, though income-sharing generally cannot replace debt as a sole source of financial aid. Even non-optimized income-sharing is shown to be an effective financial aid option for students who face intermediate levels of need.

Income-sharing in higher education began in practice with a program run by Yale University in the 1970s, which is widely regarded as having failed due to high inflation and mistakes in the program design Curran (2018). ISAs were revived at Purdue University in 2016, and have since been established at dozens of institutions. Some universities, such as Purdue and the University of Utah offer income-sharing with the help of companies, including Vemo Education, that specialize in providing income-share agreements. Other universities, such as Clarkson University and Colorado Mountain College, provide ISAs directly through the university with funds raised from donors. Others still, such as the University of California, San Diego, have partnered with local businesses that specialize in helping students in particular fields find good jobs, like the San Diego Workforce Partnership. ISAs of the latter two types are often provided only to students who major in specific fields, and the funding provider works more closely with the students; the last type in particular invests heavily in the labor market success of partnered students. Analyzing the labor-market outcomes of modern income-share agreements empirically remains infeasible, due to the only very recent roll-out of these programs; few students with ISAs have joined the labor force. Until such empirical work is possible, I seek to find the impacts of income-sharing through a straightforward labor search model. Job applicants draw from a wage distribution at a particular rate, and choose either to accept a wage offer or return to searching. By stripping the model down to only the bare essentials, I am able to isolate the effects of income-sharing on the outcomes of interest. When an applicant has an ISA, she does not get to keep the entirety of the offered wage; if an applicant has an ISA provider who invests in them, she receives job offers faster. This comes from thinking of investment as helping an applicant by placing calls on her behalf to potential employers, helping write résumés and cover letters, or teaching interviewing techniques. With only these factors distinguishing income-sharing from traditional lending in the model, the results are broadly applicable. The impact of ISAs I find are distillations of only the fundamental features of income-share agreements.

Related literature

This paper contributes to a number of active research areas. Most crucially, I add to the small number of vital studies of income-share agreements, beginning with Madonia and Smith (2019), who study the impact of ISAs on tournament poker players' effort, showing empirically that players' returns fall substantially when they sell stakes in their winnings, which can be attributed both to selection into tournaments with more talented opponents and to a reduction in the incentives to play at one's best. The study of ISAs with in an educational context begins with the recent working paper Mumford (2018). Mumford uses data from Purdue University to study what drives selection into ISAs, finding that parents' socioeconomic conditions is the largest factor in students' funding choices, and that, within a given major choice, students with lower grades or aptitudes do not adversely select into ISAs. Furthermore, Mumford shows that ISA enrollment is not driven by students with greater risk-aversion. Like Madonia and Smith, I look at the outcomes of ISA-funded individuals compared with the outcomes of traditionally funded individuals. Like Mumford, I consider educational ISAs and students' choices of funding options. Inspired by his results, the financial background of the students is the primary driver of students' decision calculus in my model. A comparison between ISAs and debt in higher education is the subject of the working paper Balakrishnan and Cynamon (2018), which tries to compare debt to ISAs for a student's overall college value premium by constructing a synthetic panel of graduates, and finds that ISAs reduce the average student's lifetime college value premium by a much

smaller amount than does debt. Nonetheless, I am the first to develop a framework that includes selection into an ISA with long-run outcomes. In particular, I am the first to show wage and employment effects of educational ISAs, as insufficient data currently exists for empirical work, given the nascence of income-sharing in the world of higher education.

I also contribute to a literature on optimal student lending design. Recent work in this area, such as that by Britton et al. (2019), Britton and Gruber (2019), and Lochner and Monge-Naranjo (2016), have shown the value of income-contingent loan repayment options for students. While ISAs are not loans, this paper shows that well-designed income-sharing agreements can be preferable to debt under broad circumstances, and can be a part of the optimal college financing landscape.

I adapt techniques from the literature on labor search. Classic papers such as McCall (1970), Mortensen (1970), Gronau (1971), and Lippman and McCall (1976) along with expansions on their models in Mortensen (1986) and Mortensen and Pissarides (1999) serve as the theoretical underpinnings of my model in this paper. I use the McCallian framework, rather than the benchmark Diamond (1982) and Mortensen and Pissarides (1994) model, in order to isolate the impact of ISAs on search behavior itself. This is particularly important as ISAs are newly introduced to a market environment that has long since reached maturity in their absence, while job applicants with ISAs are rare relative to those who are traditionally financed.

In the following section, I lay out a model of labor search that incorporates income-sharing and further enhance it with an investment technology for ISA providers. Section 3 analyzes the effects of ISAs on various measures of labor market success and access to college. Students are allowed to select their methods of financing in section 4, in which student and social welfare is discussed, followed by the construction of the optimal income-share agreement. Section 5 concludes.

2 Model

Job search without income-sharing

A risk neutral job applicant discounts time at rate r in continuous time across an infinite horizon¹. She begins time unemployed, and receives a flow payoff b as long as she remains unemployed. The value b > 0 may be thought of as the flow value of the leisure she enjoys by not having a job. At the beginning of time, she also carries debt Δ , which will be endogenized

 $^{^{1}}$ The analysis in this subsection is directly adapted from McCall (1970) and the related literature described above.

later.

As long as the applicant is unemployed, she draws a wage offer i.i.d. at rate $\alpha > 0$ from an exogenous, stationary distribution F(w) with bounded support. Whenever she draws an offer, she has the choice to accept the offer, earning the drawn wage as a flow payoff forever², or to reject the offer and return to searching for a job. The value to the applicant of accepting an offer w is

$$W(w) = \frac{w}{r} - \Delta. \tag{2.1}$$

As the value of accepting a wage offer is increasing in the offered wage, the applicant follows a threshold strategy with reservation wage w_R and the leisure and search value of unemployment must be the value of accepting a job with at the reservation wage, i.e. $U = W(w_R)$. While searching for a job, the applicant enjoys not only the flow value b but additionally the option value of being able to search for a permanent job. Thus, the applicant's flow value of unemployment can be written

$$rU = b + \alpha \int_0^\infty \max\{0, W(w) - U\} \, dF(w).$$
(2.2)

At any time before accepting a job, the applicant is earning b, and with probability α gets to choose between searching more, or accepting the value of the drawn wage offer and forgoing further search.

Combining the equations defining the value functions of employment and unemployment, we get the classical reservation wage equation

$$w_R = b + \frac{\alpha}{r} \int_{w_R}^{\infty} (w - w_R) \, dF(w) = b + \frac{\alpha}{r} \int_{w_R}^{\infty} (1 - F(w)) \, dw.$$
(2.3)

The right-hand side of this fixed-point equation is easily shown to be a contraction, and thus there is a unique fixed-point that can serve as the applicant's reservation wage.

It is important for us to note that the optimal search strategy of the applicant is independent of Δ , the debt that the applicant carries, as the debt she carries must be repaid regardless of her employment status.

Job search with income-sharing

We now turn to our main consideration of income-share agreements. An applicant who has accepted an ISA, receiving an amount Δ^E in return for a share ϕ of her future income, is

 $^{^{2}}$ This assumption does not impact the qualitative results of the analysis, in this or in later sections, provided that the wage distribution remains fixed.

broadly similar to the traditional debt-financed student analyzed above. However, the value to a student with an ISA of accepting a wage offer w is

$$W(w) = (1 - \phi)\frac{w}{r} - \Delta + \Delta^E.$$
(2.4)

as she must give a fraction of her income to the ISA provider, and has received some upfront payment from the ISA provider.

As before, the value of accepting a wage increases with the wage itself, so a threshold strategy must be optimal. Following the same derivation process, we arrive with a similar reservation wage equation for an ISA-financed student:

$$(1-\phi)w_R^E = b + (1-\phi)\frac{\alpha}{r} \int_{w_R^E}^{\infty} (1-F(w)) \, dw.$$
(2.5)

From this we immediately arrive at the following lemma by implicitly differentiating the reservation wage w_R^E with respect to the income-sharing rate ϕ .

Lemma 2.1. The reservation wage is increasing in the income-sharing rate.

As the percentage of her income that an applicant must relinquish to the ISA provider increases, the value of working relative to unemployment decreases. The wage that made her indifferent between accepting and declining a job offer is now insufficient, and she prefers to wait for a better offer, even though waiting itself is costly.

Investment in the contact rate

Unlike traditional lenders, income-share providers have an incentive to increase a funded applicant's wages if it is possible to do so. ISA providers are paid more when the students they fund get higher-paying jobs, and they get paid more when the students they fund get jobs sooner, since they discount the future. Providers should look for any possible avenue with which they can ease the search process, and evidence from the San Diego Workforce Partnership suggests that they do so by assisting applicants with résumé writing and helping applicants access a network of potential employers in relevant fields. Using this stylized fact as inspiration, we think of the provider as being able to make costly investments in an applicant's rate of receiving job offers.

Rather than receiving wage offers at rate α , an investment-type ISA funded applicant receives offers at rate $\alpha^N \geq \alpha$, chosen by the ISA provider. The provider faces a cost $c(\alpha^N)$ described by the assumption below:

Assumption 1. $c(\alpha^N)$ satisfies

- $c(\alpha) = 0$
- $c'(\alpha^N) \ge 0$
- $c''(\alpha^N) > 0$
- c'(0) = 0
- $\lim_{\alpha^N \to \infty} c'(\alpha^N) = \infty$

These conditions serve to ensure that the ISA always finds some unique amount of investment profitable, but is only willing to invest a finite amount.

The applicant's search problem remains similar to that derived above, with

$$(1-\phi)w_R^N = b + (1-\phi)\frac{\alpha^N}{r} \int_{w_R^N}^{\infty} (1-F(w)) \, dw$$
 (2.6)

but there is now a new condition required to define equilibrium, since the provider must choose α^N optimally. The applicant's reservation wage equation describes a unique optimal reservation wage for any given contact rate, so we can treat the provider as optimally choosing the applicant's reservation wage, subject to said reservation wage satisfying equation 2.6, and maximizing his own payoff.

Just like the applicant, we can define value functions for the provider. The value V(w) to the provider of the applicant accepting wage w needs to balance the monetary value of the wage itself with the delay caused by searching. The following lemma gives the provider's value of a particular reservation wage.

Lemma 2.2. The value to the provider of the applicant having reservation wage w_R^N is

$$V(w) = \frac{\phi w_R^N H^N}{r + H^N},$$

in which

$$H^N := \alpha^N \left(1 - F\left(w_R^N \right) \right),$$

Since the applicant starts out unemployed, and the provider is choosing the applicant's reservation wage implicitly through his own choice of α^N , the provider's value of the applicant's unemployment is equal to the value to the provider of the applicant accepting a job at the reservation wage, adjusted to account for the fact that higher reservation wages take longer to implement, net the initial cost of implementing a given contact rate. Thus, the ISA provider's

problem is to maximize

$$\max_{w_R^N,\alpha^N} \phi \frac{w_R^N H^N}{r + H^N} - c\left(\alpha^N\right), \text{ s.t. } (1 - \phi)w_r^N = b + (1 - \phi)\frac{\alpha^N}{r} \int_{w_r^N}^\infty (1 - F(w)) \, dw.$$
 (2.7)

The ISA's solution to this problem defines a system of two equations which characterize labor-search under an investment-type ISA. Equation 2.6 and

$$c'\left(\alpha^{N}\right) = \phi w_{R}^{N} \frac{1 - H^{N}}{r + H^{N}} \frac{\partial H^{N}}{\partial \alpha^{N}} + \phi \frac{H^{N}}{r + H^{N}} \frac{\partial w_{R}^{N}}{\partial \alpha^{N}}$$
(2.8)

together characterize the ISA provider's optimal choice of contact rate and the applicant's choice of reservation wage. The analysis of the optimal choices of w_R^N and α^N are simpler when the partials of H^N and w_R^N with respect to α^N are not written out in full, so they are not written out here.

The following lemma, comparing investment-type ISAs to non-investment ISAs and traditional student loans, must hold, following from assumption 1 and the applicant's search condition 2.6.

Lemma 2.3. If an ISA is able to invest in job applicants, it will do so, and the reservation wage of an applicant who receives investment will be higher than the reservation wage of an applicant with an ISA who does not receive an investment, which is higher again than an applicant funded through traditional loans.

- $\alpha^N > \alpha$
- $w_R^N > w_R^E > w_R$

The very goal of investment is to increase the reservation wage of the job applicant, so as long as assumption 1 holds, investment will be forthcoming and gross wages will be under for investment-ISAs.

An increase in the income-sharing rate increases the reservation wage for a non-investment ISA, since the flow value of remaining unemployed is relatively more valuable when the value of accepting a given wage decreases. It is less immediate, however, that increasing income-sharing would have a similar effect for investment-type ISAs. The income-sharing rate not only impacts the optimal choice of reservation wage, but also the optimal investment in the contact rate. Increases in the contact rate serve to increase reservation wages, but it may not always be so that increases in income-sharing increase the contact rate. Nonetheless, it is still the case that increases in income-sharing serve to increase the reservation wage even in investment-type ISAs.

Proposition 2.1. The reservation wage w_R^N of an investment-type ISA funded applicant is increasing in the income-share ϕ .

The intuition for this proposition is straightforward. The reservation wage is increasing in the contact rate, as increasing the contact rate directly increases the option value of continuing to search for more jobs. Since an increase in the income-sharing rate, holding the contact rate fixed, also increases the relative value of searching, it is only possible for an increase in the income-sharing rate to decrease the overall reservation wage if the increase in the income-share *decreases* the contact rate chosen by the provider. The provider would like to maximize the applicant's reservation wage given his cost function and the optimality condition for the applicant. Since changing the income-sharing rate does not change the investment cost, the provider would only decrease the contact rate if the reservation wage chosen by the applicant goes up to such an extent as to outweigh the benefits from reducing the contact rate and saving money. That is, the provider would reduce investment when income-sharing rises only in circumstances in which the reservation wage increases anyway.

3 Labor-market outcomes and access to college

The effect of an income-share agreement on labor-market outcomes extends far beyond the reservation wage chosen by job applicants that have been analyzed thus far. Before considering the welfare implications of ISAs, it is instructive to consider the impact of income-share agreements on specific goals that may be of interest to policy-makers.

Take-home pay

Most immediately, the take-home pay of an employee who has signed an ISA, unlike that of an employee without one, is not the wage paid by the employer, but rather a fraction thereof. While ISA-funded job applicants have been shown to have a higher reservation wage than those without ISAs, they may end up with a lower net wage after a portion of their pay is diverted to the ISA provider.

Depending on the draw from the wage distribution that induces an applicant to stop searching and begin employment, an employee with or without an ISA may have a higher take-home pay simply through luck. Therefore, attention is focused on the take-home portion of the applicant's *reservation* wage, rather than her observed wage, as, *ex ante*, this is the income-level that is most closely associated with the applicant's actual welfare. This is the flow value of take-home pay that makes the applicant indifferent between taking the job or continuing to search. It is reasonable to expect that, as income-sharing becomes more extreme, take-home pay would decrease. In the limit as the provider gets to keep nearly all of the earned wages, the boundedness of the wage distribution prevents the applicant from simply waiting for a wage offer sufficiently high so as to make up for the lost income, and all value created by a successful search flow to the provider. In fact, when investment is not an option for the provider, an increase in income-sharing will *always* result in a decrease in take-home pay.

Lemma 3.1. The applicant's take-home pay in a non-investment ISA, w_R^E , is decreasing in the income-sharing rate ϕ .

This is easily shown by taking the derivative of the right-hand side of equation 2.5 with respect to ϕ :

$$\frac{\partial(1-\phi)w_R^E}{\partial\phi} = \frac{-\alpha}{r} \left[(1-\phi)\left(1-F\left(w_R^E\right)\right) + \int_{w_R^E}^{\infty} 1 - F(w)\,dw \right] < 0.$$
(3.1)

Since the reservation wage itself is increasing in ϕ , take-home pay is decreasing. By increasing income-sharing, the provider is increasing wages, but is additionally capturing rents from the applicant herself.

However, this bleak result does not carry over into the scenario in which ISA providers invest in applicants.

Proposition 3.1. Introducing income-sharing increases take-home pay.

By investing in the contact rate of the applicant, the provider increases the reservation wage of the applicant. Increasing the income-sharing rate, then, may cause the reservation wage to increase through two channels—directly, as jobs are less valuable when income-sharing is in effect, and, if the income-sharing has also increased the provider's investment, indirectly by making searching more valuable. The direct effect, we have seen, is insufficient for raising take-home pay, but the contact rate rises enough for the provider to bring enough extra value to the search process, allowing for take-home pay to increase.

A note about investment

In an investment-type ISA, it becomes possible for take-home pay to increase precisely because an option is available to the provider and applicant that is not available through other types of financing. It is natural, then, to consider whether gains from investment are truly a feature of income-share agreements, or rather if investment itself is the proper focus of our attention. A number of points may be raised in defense of the author's viewpoint that investment as a byproduct of the ISA itself, rather than investment in its own right, is the proper focus of study.

When considering the traditional debt-financed job applicant, one would be hard pressed to conclude that the government, through its student lending process, provides students with an explicit advantage in the job search process in practice, nor indeed does any private lender. ISAs, however, have in some circumstances been provided by organizations who provide applicants with résumé assistance and direct access to employer networks. In the world as we observe it, ISAs are in fact investing in applicants in a way that traditional funding sources are not. Indeed, it hardly should be surprising that ISAs invest while traditional lenders do not. Were a traditional lender to be endowed with the same investment technology as an ISA, there would be no incentive for them to use it. Student loan debt is nondischargable through bankruptcy, and thus most borrowers generally are obliged to pay back the same amount of money regardless of their income. Even under income-contingent loan repayment plans, the total amount the borrower is required to repay is capped by the principal of the loan, plus interest, leaving the potential upside to the lender of investing substantially lower than for an ISA provider.

It is possible, too, that job applicants are able to invest on their own in the contact rate, regardless of their method of funding. Services that charge job applicants for the same kinds of improvements to their searching abilities offered by ISAs abound. However, it is likely, that ISA providers may be able to invest in the search process at a cheaper cost than the applicants themselves can, given that they may specialize in such activities, but it is admittedly the case that a student who keeps her entire earnings is more incentivized to invest in her labor-market outcomes than one who keeps only a portion of her wage for herself. However, as long as ISAs can provide investment services more efficiently than students have external access to, or if ISA investment can complement student-led investment rather than substitute away from it, the incentive for such investment is so uniquely aligned between ISA and student that it is impossible to separate the investment from the ISA for the sake of analyzing the overall effects of income-sharing on labor-market outcomes.

Unemployment

A policy-maker may also be interested in the effects of ISAs on unemployment. In our model of the job market, unemployment is frictional. Individuals are unemployed because the search process is not instant; it takes time for applicants to find jobs that they are willing to accept. We have seen that take-home pay is lower among ISA-funded applicants who do not receive investment, and that it may be lower among even those ISA-funded applicants who do. If finding a job takes longer for such an applicant, only to result in a lower take-home pay, then the additional frictional unemployment can be interpreted negatively for the applicant, even if it may potentially be positive overall in conjunction with the provider's share of income. Furthermore, when investment costs are such that the take-home wage of the applicant increases under an ISA, an increase in frictional unemployment does not necessarily end in a worse labor-market outcome for applicants. In view of this, the author does not seek to claim that increases or decreases in unemployment are necessarily good or bad, but instead to simply describe the effects of ISAs on unemployment.

The probability that a debt-financed applicant has not found a job by time t is e^{-Ht} , in which $H := \alpha (1 - F(w_R))$ is the hazard rate of the search process, equal to the probability in a given instant of receiving a wage offer greater than the reservation wage. Similarly for ISA-funded applicants, we can define $H^E := \alpha (1 - F(w_R^E))$ and $H^N := \alpha^N (1 - F(w_R^N))$. The expected length of time until the applicant finds a job, or the expected time for them to remain unemployed, is simply the inverse of the hazard rate, $\frac{1}{H}$, $\frac{1}{H^E}$, or $\frac{1}{H^N}$. An increase in the hazard rate can be thought of as an increase in the speed of finding a job, or equivalently a decrease in unemployment.

Non-investment type ISAs lead to longer unemployment spells than do traditional student loans, and increases in income-sharing lengthen the average time until an applicant finds a job, as shown in the following lemma.

Lemma 3.2. Increasing the income-sharing rate decreases the hazard rate H^E , i.e., $\frac{\partial H^E}{\partial \phi} < 0$.

The intuition behind this result is straightforward; increasing income-sharing increases the reservation wage of an applicant, making any given drawn wage offer less likely to be accepted. Since the contact rate is being held constant, it takes longer on average for an applicant to find a job that pays sufficiently well, and she remains unemployed longer.

Here we see that for ISA-funded applicants who do not have investments from their provider, it takes longer to find a job on average than a debt-financed applicant, and the job that eventually gets accepted has a lower expected take-home pay on average than that of a debt-financed applicant.

Things are more complicated investment-type ISA funded applicants, however, as the speed of search must decrease when income-sharing is high, though it is also possible for the speed of search to increase, as shown in the next proposition.

Proposition 3.2. Both of the following properties hold for investment-type ISAs:

• $\lim_{\phi \to \overline{\phi}} H^N = 0$, in which $\overline{\phi} = 1 - \frac{b}{\overline{w}}$ and \overline{w} is the supremum of the support of the wage distribution

• $\lim_{\phi \to 0} \frac{\partial H^N}{\partial \phi} > 0.$

As the income-sharing rate approaches the point for which the applicant would be as well off collecting just the flow value of unemployment as she would be earning the highest possible wage, the applicant's reservation wage approaches this maximal wage \overline{w} ; there is no incentive for her to take a job that gives her a lower take-home pay than the flow value of unemployment. The probability that any drawn wage offer is higher than her reservation wage approaches zero. However, by assumption 1, the provider is unwilling to increase investment fast enough to keep the hazard rate from collapsing to zero, due to the convexity of the cost function. Thus, in the limit, the applicant never finds a job and remains unemployed forever.

When the income-sharing rate is low, however, it may be possible to have increases in the income-sharing rate actually *increase* the speed of search. When income-sharing is just introduced, the increase in the contact rate speeds up search more than the increase in the reservation wage can slow it down, and unemployment drops as ϕ rises.

College access

College access may not be a labor-market outcome, but is also potentially affected by the presence of ISAs. Given that ISAs supplement the traditional student loan offerings, rather than replacing them, adding ISAs as an option for students cannot reduce college access. However, under some circumstances, the number of students who are able to afford university educations may increase.

Suppose the net cost of college for a given student is K. This cost represents the amount that the student needs to finance through debt or ISAs to meet tuition, fees, and room and board costs, and does not include any money that the student or her family may have saved previously, nor does it include any scholarship or grant money that she may have been awarded. This value varies, then, among even students who attend the same college and who have the same course of study and job-market prospects.

A student is willing to attend college if and only if the cost of college is less than the value of attending. In our model, the value of attending college is exactly the unemployment value of a job applicant, i.e., the value of the stream of take-home pay that the student is minimally willing to accept. Thus, the student will attend college whenever $K < \frac{w_R}{r}$ if she has access to traditional student debt options.

When students have access to ISAs, the value of attending college changes along with the wage prospects. As discussed above, take-home pay goes down with ISAs that do not have investment, so $\frac{w_R^E}{r} < \frac{w_R}{r}$, and students with high net costs of college $K \in \left((1-\phi)\frac{w_R^E}{r}, \frac{w_R}{r}\right)$ are willing to attend college *only* through traditional debt, and would not attend if ISAs were

the only available method of financing their education.

However, when ISAs have opportunities for investment, and investment costs are low, it is possible for $(1 - \phi)w_R^N > w_R$. In this scenario, the existence of ISAs allows students who would ordinarily not attend college due to high costs to find college newly valuable. For $K \in \left(\frac{w_R}{r}, (1 - \phi)\frac{w_R^N}{r}\right]$, students are only willing to attend college with ISAs, and the existence of ISAs with investment increases the number of students who have access to higher education. Because K, the net cost of college of a given student, is likely to be highest among students with the lowest socioeconomic status, investment-type ISAs may provide the greatest benefits to those who have the worst opportunities without them.

4 Optimal policies

The analysis to this point takes the choice of financing method as given. In this section, the choice of a student to accept an income-share agreement is endogenized, allowing an understanding of who is likely to benefit from ISAs and to design both the optimal incomeshare agreement for students and the optimal income-share agreements for providers.

At any given income-sharing rate, a student who chooses to accept an ISA of any amount should opt to accept as much money as the provider is willing to give. Conditional on the percentage of income offered to the provider, the repayment amount that a student is obligated to provide is independent of the amount received upfront, and both the initial sum provided by the ISA and the balance of any debt that the student may have are irrelevant to labor-market outcomes, so there is no reason for the student to turn down money offered to them. It is of vital importance, then, to ask how much money an ISA provider is willing to contribute to any given student. *Ex-ante*, the provider values an income-share agreement at $\phi \frac{w_R^E H^E}{r+H^E}$ if they cannot invest, and at $\phi \frac{w_R^N H^N}{r+H^N} - c(\alpha^N)$ if he can invest. Thus, a non-investment type ISA provider is willing to provide up to

$$K^E := \phi \frac{w_R^E H^E}{r + H^E} \tag{4.1}$$

and an investment-type ISA provider is willing to provide up to

$$K^{N} := \phi \frac{w_{R}^{N} H^{N}}{r + H^{N}} - c\left(\alpha^{N}\right).$$

$$(4.2)$$

When should a student prefer an ISA to debt? The preference over funding options depends on several factors, primarily, the income-sharing rate ϕ and the student's individual net cost of college K. The overall choices students make are characterized in the following

two propositions.

Proposition 4.1. When ISAs do not include investment, a student's choice of financing is characterized by the following. For $\phi \in [0, .5]$,

$$\begin{cases} K \leq \frac{w_R - (1-\phi)w_R^E}{r} & Debt \ only \\ K \in \left(\frac{w_R - (1-\phi)w_R^E}{r}, K^E\right] & ISA \ only \\ K \in \left(K^E, \frac{(1-\phi)w_R^E}{r}\right] & Maximal \ ISA, \ remainder \ debt \\ K \in \left(\frac{(1-\phi)w_R^E}{r}, \frac{w_R}{r}\right] & Debt \ only \\ K > \frac{w_R}{r} & No \ college. \end{cases}$$

For $\phi \in (.5, \overline{\phi}]$, the choice is instead characterized by

$$\begin{cases} K \leq \frac{w_R - (1-\phi)w_R^E}{r} & Debt \ only\\ K \in \left(\frac{w_R - (1-\phi)w_R^E}{r}, \frac{(1-\phi)w_R^E}{r}\right] & ISA \ only\\ K \in \left(\frac{(1-\phi)w_R^E}{r}, \frac{w_R}{r}\right] & Debt \ only\\ K > \frac{w_R}{r} & No \ college. \end{cases}$$

Students prefer ISAs if the take-home pay that they forfeit through a worse labor-market outcome is less than the amount of debt that they forgo by accepting the ISA. However, there are two problems that prevent ISAs being able to simply replace debt for high-cost students: providers are only willing to provide up to K^E , and the overall value of the labor-market is lower for ISA-funded students than debt-financed students, so the highest-cost individuals *cannot* benefit from ISAs without investment.

When income-sharing is particularly extreme, providers are willing to provide a larger sum of money than the student herself takes home, so the scenario in which the student fully exhausts ISA funds but still requires debt to meet the cost of college ceases to exist.

When an ISA is able to invest, then there are some ϕ for which take-home pay is larger with the ISA than without, so students always prefer funding with an ISA as much as possible.

Regardless of whether or not investment is possible, income-share agreements are chosen by some students. In particular, even when investment does not increase take-home pay, all students facing a cost of college between $\frac{w_R - (1-\phi)w_R^E}{r}$ and $\frac{(1-\phi)w_R^E}{r}$ use ISAs to some degree in their optimal financing package. Furthermore, we can look at how changing the income-sharing rate affects how many students choose ISAs.

Proposition 4.2. For non-investment type ISAs, the measure of the interval of costs such

that students use an ISA decreases for all $\phi \in (0, \phi]$.

As ϕ increases, debt becomes a relatively better alternative to ISAs, increasing the minimum cost of college necessary before a student chooses an ISA. The overall value of college decreases with increases in the income-share, so some partially-ISA funded students are required also to substitute away towards debt, causing the set of students who fund using any ISA to shrink overall as ϕ goes up.

The optimal ISA

Now that the effects of income-share agreements have been fully characterized, along with the scenarios under which students elect to use them, it is possible to design the optimal ISA. It is impossible to consider the optimal income-sharing rule ϕ on its own, as changing ϕ changes whether or not a given student accepts an ISA, so it is important to balance the intensive-margin welfare effects of changes in income-sharing with extensive-margin effects of students entering or leaving behind ISAs.

A policy designer maximizing student welfare or one maximizing provider welfare wants to choose the ideal income-sharing rate *conditional on the student choosing an ISA*. A student elects to use an ISA only when ISAs provide her with higher welfare than does traditional debt. Since changing ϕ does not affect the welfare implications of the debt-financed labor market, if any ϕ can cause the applicant to use an ISA, the student's welfare is necessarily higher than it would have been under debt-only financing. Similarly, ISA providers only earn revenue if students use their services.

Indeed, if a ϕ can be chosen that allows a student to fully-fund her education with an ISA, fully funding through an ISA must be the student-welfare optimizing choice.

Proposition 4.3. Let

$$w^* = 2b + \frac{\alpha}{r} \int_{w^*}^{\infty} 1 - F(w) \, dw.$$

Then, when investment is not possible,

- Any K such that $K \leq \frac{w^*}{2r}$ can be fully funded by an ISA, and is in the optimal ISA
- For any such K, the optimal ϕ for students satisfies $K = K^E$
- For any such K, the optimal ϕ for providers satisfies $rK = w_R (1 \phi)w_R^E$
- The student-optimal ϕ is lower than the provider-optimal ϕ
- For $K \in \left(\frac{w^*}{2r}, \frac{w_R}{r}\right)$, the optimal ϕ for students satisfies $rK = (1 \phi)w_R^E$

• For $K \in \left(\frac{w^*}{2r}, \frac{w_R}{r}\right)$, the provider is indifferent between all possible choices of ϕ .

When fully funding education through an ISA, the student-optimal income-sharing rate is the lowest rate required to allow full funding, while the provider-optimal income-sharing rate is the highest such rate. On the other hand, if it is impossible to fully fund through an ISA, the student actually prefers a high income-sharing rate. The student has to borrow any remaining cost after the payment she receives from the ISA provider, and the provider is willing to give the student up to the amount she expects to receive from the student in the future. Thus, a student who fully exhausts her ISA options can be thought of as receiving the entire reservation wage, rather than just the take-home portion, as the portion provided to the provider directly corresponds to debt the student does not need to take out. Increasing the income-sharing rate increases the reservation wage, so the student-optimal income-sharing rate would be the one that maximizes the reservation wage while ensuring that the student is able to earn enough to fully pay for her education.

5 Conclusion

Income-share agreements have grown in importance in higher education over the last several years, but they remain understudied. In this paper, I have modeled how income-share agreements can affect labor-market outcomes through several different measures, and have shown that these agreements have far reaching impacts that differ from traditional lending in significant ways. As ISA providers continue to expand nationwide, it will be important for regulators, financial aid officers, parents, and above all students to understand ISAs' differences from student debt, so that students can be assured of their individually best options.

My results show that ISAs may vary by type, but that the most common form of ISAs is likely to result in lower take-home pay and longer job searches for the average job applicant. Importantly, ISAs that invest in job applicants can help applicants both find jobs faster and find jobs that pay more than their debt-financed peers. This alignment between the incentives of provider and student is a valuable aspect of the ISA environment, and it would behoove both regulator and student to approach investment-type ISAs differently from those ISAs that do not invest in applicants.

Even for income-share agreements that do lower take-home pay or decrease the speed of finding jobs, students often can benefit through a reduction in the size of their traditional loan obligations. In fact, optimally-chosen personalized ISAs are part of the optimal financial-aid package for every student who wishes to attend college, with students who need relatively little aid being able to fully finance their education with ISAs and students with the greatest need being able to partially substitute away from debt. For ISAs *with* investment, students may prefer standardized, rather than personalized, ISAs to debt even when their need is very low, and students with the greatest need may be able to access higher education despite being unable to do so before. ISAs of all types can be beneficial to have in the financial-aid toolkit, but the most helpful ISAs are the ones that come with the most focus on the individual needs of the student, providing optimally chosen, personalized income-sharing rates or direct investment in the job-market process.

There are likely effects of ISAs that are left out of the analysis in this paper. All decisionmakers in this model are risk-neutral, which means that the impact of different risk attitudes among students on selection into ISAs is not discussed. It is reasonable to expect that greater levels of risk aversion would be associated with higher take-up of ISAs relative to debt, since income-share agreements offer a type of insurance against bad labor-market outcomes that is missing from most debt repayment plans. Such a mechanism would suggest that my results likely undervalue the usefulness of ISAs as an option students could benefit from. On the other hand, ISAs may attract adverse selection and moral hazard, in that students who expect to be less productive employees may see ISAs as particularly cheap pathways to affording higher education or that students who have ISAs may put less effort into their schoolwork, their job search, or their job in light of the fact that they do not get to take home the full value of their productivity. This may make firms less willing to hire an ISA-funded applicant at any given wage. While my model cannot capture this dynamic, I conjecture that a reduction in the wage a firm is willing to pay for an ISA funded student would reduce the usefulness of ISAs in serving as effective financial aid. A more thorough treatment of income-sharing including the effects of moral hazard and adverse selection is left for future research.

A Proofs for section 2

Proof of lemma 2.2. The probability that the applicant has not gotten a job by time t is $e^{-H^N t}$, and so the probability that the applicant gets a job at exactly time t is equal to $H^N e^{-H^N t}$. Thus, the value to the provider of a wage w can be written

$$V(w) = H^N \int_0^\infty e^{-(r+H^N)t} \phi w \, dt.$$

Solving this integral yields

$$V(w) = \frac{\phi w H^N}{r + H^N}.$$

Proof of proposition 2.1. Start by transforming 2.6 and 2.8 with $\gamma := 1 - \phi$. Then, we take the γ -derivative of these first order equations. Rearranging, we get the system

$$\frac{\partial w_R^N}{\partial \gamma} \left(1 + \frac{\alpha^N \left(1 - F\left(w_R^N\right) \right)}{r} \right) = \frac{-b}{\gamma^2} + \frac{1}{r} \int_{w_R^N}^{\infty} 1 - F(w) \, dw \, \frac{\partial \alpha^N}{\partial \gamma} \tag{A.1}$$

$$\frac{\partial \alpha^{N}}{\partial \gamma} \left(c'' \left(\alpha^{N} \right) \left(r + H^{N} \right) + c' \left(\alpha^{N} \right) \left(1 - F \left(w_{R}^{N} \right) \right) \right) = \frac{-c' \left(\alpha^{N} \right)}{1 - \gamma} + (1 - \gamma) \left[\left(1 - H^{N} \right) \frac{\partial H^{N}}{\partial \alpha^{N}} \frac{\partial w_{R}^{N}}{\partial \gamma} - w_{R}^{N} \frac{\partial H^{N}}{\partial \alpha^{N}} \frac{\partial H^{N}}{\partial \gamma} + w_{R}^{N} \left(1 - H^{N} \right) \frac{\partial^{2} H^{N}}{\partial \gamma \partial \alpha^{N}} + \frac{\partial w_{R}^{N}}{\partial \alpha^{N}} \frac{\partial H^{N}}{\partial \gamma} + H^{N} \frac{\partial^{2} w_{R}^{N}}{\partial \gamma \partial \alpha^{N}} \right] \quad (A.2)$$

First, consider the point for which $\phi = 0$, or $\gamma = 1$, and so $w_R^N = w_R$ and $\alpha^N = \alpha$. At this point, lemma 2.3 implies that both $\frac{\partial w_R^N}{\partial \gamma}$ and $\frac{\partial \alpha^N}{\partial \gamma}$ are negative; when income sharing is introduced, the reservation wage and the contact rate both go up. The reservation wage must be continuous in ϕ , and equivalently in γ , so by the intermediate value theorem, if $\frac{\partial w_R^N}{\partial \phi} < 0$ ever holds, there must be some $\overline{\gamma}$ for which $\frac{\partial w_R^N}{\partial \gamma}\Big|_{\gamma=\overline{\gamma}} = 0$. At this point, since by the chain rule, $\frac{\partial w_R^N}{\partial \gamma} = \frac{\partial w_R^N}{\partial \gamma} \frac{\partial \alpha^N}{\partial \gamma}$, it must be that $\frac{\partial \alpha^N}{\partial \gamma}\Big|_{\gamma=\overline{\gamma}} = 0$, and thus also $\frac{\partial H^N}{\partial \gamma}\Big|_{\gamma=\overline{\gamma}} = 0$. Simplifying, then, we get

$$0 = \frac{-b}{\overline{\gamma}^2}$$
$$\frac{c'\left(\alpha^N\right)}{1-\gamma} = (1-\gamma) \left[w_R^N \left(1-H^N\right) \frac{\partial^2 H^N}{\partial \gamma \partial \alpha^N} + H^N \frac{\partial^2 w_R^N}{\partial \gamma \partial \alpha^N} \right]$$

The first line yields a contradiction, as γ is bounded. Since the reservation wage is increasing in ϕ when $\phi = 0$, and the reservation wage must be continuous in ϕ , it must be that the reservation wage is *always* increasing in ϕ .

B Proofs for section **3**

Proof of proposition 3.1. In order to ensure that a local increase in take-home pay yields a strictly greater take-home pay than traditional debt financing, we check for a condition local to $\phi = 0$. Again, let $\gamma := 1 - \phi$. Further, let $H := \alpha (1 - F(w_R))$ be the hazard rate of the search process at $\phi = 0$. Then, assuming that the partials of w_R^N and α^N are bounded and using L'Hôpital's Rule when necessary, we find that

$$\frac{\partial w_R^N}{\partial \gamma}\Big|_{\gamma=1} (r+H) = -rb + \int_{w_R}^{\infty} 1 - F(w) \, dw \, \left. \frac{\partial \alpha^N}{\partial \gamma} \right|_{\gamma=1} \\ \frac{\partial \alpha^N}{\partial \gamma}\Big|_{\gamma=1} c''(\alpha^N) \left(r+H\right) = c''(\alpha) \left. \frac{\partial \alpha^N}{\partial \gamma} \right|_{\gamma=1}$$

From the second equality, we find that either r + H = 1, which is generally false, or that $\frac{\partial \alpha^N}{\partial \gamma} = 0$ whenever $\gamma = 1$, which is also false, as $\frac{\partial w_R^N}{\partial \gamma} > 0$ and the chain rule requires the partials of w_R^N and α^N to have the same sign. Our assumption that the partials are bounded at $\gamma = 1$ must then be false. In particular,

$$\lim_{\gamma \to 1} w_R^N = \lim_{\gamma \to 1} \alpha^N = -\infty.$$

Since we can write

$$rac{\partial \gamma w_R^N}{\partial \gamma} = \gamma rac{\partial w_R^N}{\partial \gamma} + w_R^N,$$

taking the limit of both sides as $\gamma \to 1$ shows that

$$\lim_{\gamma \to 1} \frac{\partial \gamma w_R^N}{\partial \gamma} = \lim_{\gamma \to 1} \frac{\partial w_R^N}{\partial \gamma} = -\infty$$

and take-home pay always increases when income-sharing is introduced.

Proof of proposition 3.2. Again we let $\gamma := 1 - \phi$. Deriving the hazard rate with respect to γ , we get

$$\frac{\partial H^N}{\partial \gamma} = \frac{\partial \alpha^N}{\partial \gamma} \left(1 - F\left(w_R^N \right) \right) - \alpha^N f\left(w_R^N \right) \frac{\partial w_R^N}{\partial \gamma} \tag{B.1}$$

For the first claim in the proposition, we define $\overline{\gamma} := \frac{b}{\overline{w}}$ and look for the limit as $\gamma \to \overline{\gamma}$. Since $w_R^N > b$ must always hold, we see that $\lim_{\gamma \to \overline{\gamma}} w_R^N = \overline{w}$. Additionally, since $\lim_{\alpha^N \to \infty} c'(\alpha^N) = \infty$, it must be that $\lim_{\gamma \to \overline{\gamma}} \alpha^N$ and $\lim_{\gamma \to \overline{\gamma}} \frac{\partial \alpha^N}{\partial \gamma}$ are both finite. Thus,

$$\lim_{\gamma \to \overline{\gamma}} H^{N} = \lim_{\gamma \to \overline{\gamma}} \alpha^{N} \left(1 - F\left(\overline{w}\right) \right) = 0$$

For the second claim, we take first-order conditions from equations 2.6 and 2.8, yielding equations A.1 and A.2. Just as in the proof of proposition 3.1, we find $\lim_{\gamma \to 1} \frac{\partial \alpha^N}{\partial \gamma} = -\infty$. Simplifying equation B.1 with $\frac{\partial w_R^N}{\partial \gamma} = \frac{\partial w_R^N}{\partial \alpha^N} \frac{\partial \alpha^N}{\partial \gamma}$, we get that

$$\frac{\partial H^N}{\partial \gamma} = \frac{\partial \alpha^N}{\partial \gamma} \left(1 - F(w_R^N) - \alpha^N f\left(w_R^N\right) \frac{\partial w_R^N}{\partial \alpha^N} \right)$$

Since the $\lim_{\gamma \to 1} \frac{\partial \alpha^N}{\partial \gamma} = -\infty$, we can see that

$$\operatorname{sgn}\left\{\left.\frac{\partial H^{N}}{\partial \gamma}\right|_{\gamma=1}\right\} = -\operatorname{sgn}\left\{1 - F(w_{R}) - \alpha f(w_{R})\left.\frac{\partial w_{R}^{N}}{\partial \alpha^{N}}\right|_{\gamma=1}\right\} = -1.$$

C Proofs for section 4

Proof of proposition 4.1. If the student can fully fund through an ISA, an ISA is preferable to debt when E

$$\frac{(1-\phi)w_R^E}{r} > \frac{w_R}{r} - K,$$

since debt must be repaid. As long as this repayment amount is sufficiently large, a student would prefer an ISA, but can only accept an ISA for the full amount if

$$K < \frac{\phi w_R^E H^E}{r + H^E}.$$

If the student must take on debt regardless of using an ISA or not, then it must be determined if

$$\frac{(1-\phi)w_R^E}{r} - \left(K - \frac{\phi w_R^E H^E}{r + H^E}\right) > \frac{w_R}{r} - K,$$

since it is preferable to accept as much ISA funding as possible if any ISA funding is accepted, and even with the ISA the student will need to take on some debt. If the student prefers an ISA to debt, she also prefers to partially fund with the ISA than to revert to only using debt. Finally, the applicant is unwilling to use either an ISA if the cost of college is higher than the value of the job-search under an ISA, so for high enough costs of college, the student is unwilling to use an ISA but is still willing to use debt to fund college. \Box

Proof of proposition 4.2. The length of all regions of K in which students elect to use ISAs is

$$\frac{(1-\phi)w_R^E}{r} - \frac{w_R - (1-\phi)w_R^E}{r} = \frac{2(1-\phi)w_R^E - w_R}{r}$$

Since take-home pay is assumed decreasing in ϕ , this region shrinks with ϕ .

Proof of proposition 4.3. A cost of college K is fully fundable through an ISA as long as

$$rK \le \min\left\{\frac{\phi w_R^E H^E}{r + H^E}, \frac{(1-\phi)w_R^E}{r}\right\}$$

That is, an ISA can fully fund education as long as it provides both enough money to the provider that the provider is willing to pay the cost K, and that it provides enough take-home pay to the student that the student is willing to go to college at all using an ISA. Since both elements of the maximization are continuously differentiable in ϕ , the maximum of the right-hand side, representing the largest amount of money possibly fully fundable by an ISA, is at some point where $\phi < .5$. When $\phi = .5$, $w_R^E = w^*$, so some cost \overline{K} such that $r\overline{K} \leq \frac{1}{2}w^*$ is the maximal fully fundable cost. The student-optimal income-sharing rate is thus the smallest ϕ for which K remains in the fully funded region, or, from proposition 4.1, the largest ϕ for which K remains in the fully funded region, or, from proposition 4.1,

$$\sup\left\{\phi: rK > w_R - (1-\phi)w_R^E\right\}.$$

For $K > \frac{w^*}{2r}$, the student cannot fully fund her education through an ISA, and so the optimal ISA will involve partially funding through an ISA. When partially funding through an ISA, the student exhausts the amount of money the provider is willing to give, making the provider indifferent between any ISAs or not providing anything at all. Further, the student treats each additional dollar of an ISA provided as decreasing the amount she has to borrow. In this manner, the student's welfare is given by

$$(1-\phi)\frac{w_R^E}{r} - \left(K - K^E\right)$$

Since the reservation wage is increasing in ϕ , the student is best off with the highest possible ϕ such that she remains in the partial-ISA funding region, which, from proposition 4.1 and lemma 2.1, is the largest ϕ such that $rK \leq (1-\phi)\frac{w_R^E}{r}$ and $\phi \leq .5$. That $\phi \leq .5$, however, is a redundant qualification, since $\phi > .5$ implies that $rK > (1-\phi)\frac{w_R^E}{r}$.

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